

Phase classification of laboratory debris flows over a rigid bed based on the relative flow depth and friction coefficients

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The friction coefficients of debris flows over a rigid bed from several previous experiments were compiled in a preliminary investigation on the classification of phase transitions in debris flows. The collected friction coefficients were compared to the theoretical values of the friction coefficients in the relationship with the relative flow depth on the basis of sediment particle size (h/d) under various conditions. The friction coefficients of debris flows with h/d values less than 20 agreed closely with the theoretical value for boulder debris flows derived from the constitutive equations, while the friction coefficients with h/d values in the range 1000–10,000 agreed roughly with the theoretical value for turbulent water flows. The friction coefficients with h/d values of 30–300 exceeded the theoretical value for both debris and turbulent water flows. These intermediate debris flows were observed in experiments involving turbulent mud flows. However, a review of these experiments revealed that they may have included debris flows in which the turbulent structure was not well developed, and could be considered as debris flows in transition from laminar to turbulent flows. In some of the transitional debris flows, an interface dividing the flow structure into an upper turbulent-flow layer and a lower debris-flow layer was observed as reported for sediment-laden flows. The friction coefficient for transitional debris flows was modeled considering the shift of this interface. The model was able to explain the value for transitional debris flows, inferring that phase transition in debris flows from laminar to turbulent flows is induced by the shift of the interface.

1. INTRODUCTION

The characteristics of debris flows vary widely, corresponding to the wide range of natural conditions under which debris flows occur. Several studies have attempted to classify debris flows based on outward features such as magnitude [Jacob, 2005], fluidity [Hungar *et al.*, 2001], and topographic conditions [Crosta *et al.*, 1990; Prochaska *et al.*, 2008]. Classifications based on flow mechanism have also been proposed. Several debris flow models have been developed individually for *boulder debris flows* that consist mainly of a rock, gravel, and water mixture [Takahashi, 1977, 1978; Tsubaki *et al.*, 1982; Egashira *et al.*, 1989]; *sediment-laden flows* [Egashira *et al.*, 1990] or *immature debris flows* [Takahashi *et al.*, 1988] in which two layers are formed with upper water and lower debris-flow layers; and *mud flows* consisting of a mixture of fine sediment and water [Daido, 1965; O'Brien and Julien, 1988]. Mud flows are further classified into two types: those with enough extremely fine sediment particles to cause an attraction between them via an electric double layer or to reduce particle settling and mixing due to hyperconcentration [Ashida *et al.*, 1986;

Shanmugam, 1996; Whipple, 1997] and mud flows in which sediment particles are mixed as the flow descends [Arai and Takahashi, 1986, Winterwerp *et al.*, 1990]. The former viscous mud flows are often treated as Bingham fluid, and the latter as turbulent flows.

Sediment particles do not actively mix, and the particles move in a laminar fashion in boulder debris flows and viscous mud flows. The constitutive equations for boulder debris flows and viscous mud flows are derived from simple models of laminar flows that focus on the stress structure among the internal particles. Although the viscosity coefficient is sometimes used to describe the debris/mud flow motion in a simple manner [Naef *et al.*, 2006], the interparticle stresses contribute to the viscosity coefficients even in those cases. In contrast, for turbulent mud flows, the turbulent structure is considered similar to that of the turbulent flow of clear water as indicated by the von Kármán constant. The fact that both laminar and turbulent flows exist in a debris flow infers that a transition from laminar to turbulent flow takes place in the debris flow, as observed in a clear water flow. However, the mechanism and critical condition for the phase transition have not yet been explained for debris flows.

Several studies have examined phase transition in debris flows. *Takahashi* [2007] suggested that the phase transition condition is a function of relative flow depth related to sediment particle diameter (h/d), based on a simple comparison between the settling velocity for an internal sediment particle and the friction velocity. Based on observations of vertical velocity profiles, Hashimoto and *Hirano* [1995] assumed that the interface exists at an h/d value of 15, below which interparticle stresses dominate. Turbulent flow was considered to develop in the upper layer for an h/d value of 15, and debris flows were classified into laminar or turbulent flows according to the h/d value. However, neither of these studies demonstrated the actual conditions of phase transition in debris flows experimentally. In any case, an approach on the basis of the similarity law may be useful in describing the phase transition. *Miyamoto and Itoh* [2003] proposed using the Reynolds number for debris flows by evaluating the viscosity coefficient based on the constitutive equations of debris flows. However, the critical Reynolds number to indicate either the laminar or turbulent flow phases was not determined, since no experimental investigations were conducted. In the area of experimental investigations, *Arai and Takahashi* [1986] and *Hirano et al.* [1992] focused on a friction coefficient theoretically derived for a boulder debris flow and a turbulent flow. Based on a comparison between their theoretical curves and experimental data, *Arai and Takahashi* [1986] and *Hirano et al.* [1992] maintained that phase transition in debris flows occurs at an h/d value of approximately 30–100. However, since most of their experimental data were distributed around the crossing point of the theoretical lines, the results were not absolutely clear.

Many flume experiments have been conducted to validate these models in studies on the individual flow types of debris flows. Experimental data in one study are often not usable in another as the proposed models generally require many different parameters. Indices such as the friction coefficient that require few parameters can be used to compare existing experimental data across a range of studies, although the friction coefficient cannot explain detailed flow properties.

This study compiled data from previous flume experiments to compare the friction coefficients in debris flows under various conditions as a first step in classifying the phase transition.

2. MATERIALS AND METHODS

2.1 Data compilation

Suitable experimental data were selected based on the following criteria. First, flume experiments of steady and uniform flows with uniform inclination and constant water-sediment supply from the upper part were listed. Next, experiments over a rigid bed with an almost uniform particle size were selected. The data had to contain particle size, sediment concentration, and the values required to calculate the friction coefficient (i.e., mean velocity, flow depth, and flume inclination). The flux sediment concentration was used for the sediment concentration when the volumetric sediment concentration was unavailable even though the two sediment concentrations are different, especially in the case of a significant concentration profile [*Egashira et al.*, 1997].

The equivalent sediment concentration (c_e) was also calculated using the following equation [*Takahashi*, 1977; *Egashira et al.*, 1997] when the required values were available:

$$c_e = \frac{\rho \tan \theta}{(\sigma - \rho)(\tan \varphi_s - \tan \theta)} \quad (1)$$

where ρ is the mass density of water, θ is the inclination, σ is the mass density of sediment particles, and φ_s is the interparticle friction angle. Data for sediment concentrations greater than c_e were eliminated since these included cases in which sedimentation possibly occurred, and the flow was actually judged to be over an erodible bed [*Itoh et al.*, 1999].

The reason why experimental data were limited to flows over a rigid bed was the difficulty in accurately measuring flow depth over an erodible bed. In addition, sediment concentration over a rigid bed depends on the water-sediment supply, while sediment concentration over an erodible bed is determined *a priori* depending on inclination only, according to the sediment exchange between the flow phase and the bed [*Takahashi*, 1977; *Egashira et al.*, 1997].

2.2 Analysis

The experimental friction coefficient f was calculated using compiled experimental data as

$$f = \frac{2gh \sin \theta}{u_m^2} \quad (2)$$

where g is the acceleration due to gravity, h is the flow depth, and u_m is the mean velocity. The relationship between f and h/d was determined by *Arai and Takahashi* [1986], *Ashida et al.* [1988], and *Hirano et al.* [1992]. The experimental f in Eq.(2) was also compared to the theoretical curves

of f for both debris flows and turbulent flows.

The theoretical f for boulder debris flows over a rigid bed was obtained as follows. The energy dissipation (Φ) in a unit volume and time for a steady debris flow is equal to the external energy supplied:

$$\Phi = \rho_m g u \sin \theta \quad (3)$$

where ρ_m is the mass density of the debris flow and u is the velocity. Integrating Eq.(3) from the bed to the surface and substituting the result into Eq.(2) results in the following equations:

$$\int_0^h \Phi dz = \int_0^h \rho_m g u \sin \theta dz = \rho_m g h u_m \sin \theta \quad (4)$$

$$f = \frac{2gh \sin \theta}{u_m^2} = \frac{2}{\rho_m} \frac{1}{u_m^3} \int_0^h \Phi dz \quad (5)$$

Here, Φ can be rewritten as follows, based on the constitutive equations for debris flows proposed by Egashira et al. [1997]:

$$\Phi = K(c)d^2 \left(\frac{\partial u}{\partial z} \right)^3 \quad (6)$$

$$K(c) = \frac{1}{\alpha} k_g \sigma e^2 c^{\frac{1}{3}} + k_g \sigma (1 - e^2) c^{\frac{1}{3}} + \rho k_f \frac{(1 - c)^{5/3}}{c^{2/3}} \quad (7)$$

where d is the particle size, α is the ratio of the internal pressures expressed as a power function of c , k_g is an empirical constant of 0.0828, e is the coefficient of restitution, and k_f is a parameter of the disorder scale of a particle gap in the range

0.16–0.25, which is determined experimentally [Itoh and Egashira, 1999]. In this study, a value of $k_f = 0.08$ was used based on the work of Suzuki et al. [2003], which suggested the excess evaluation of Reynolds stress calculated with a value of 0.16–0.25.

A constant value is assumed for c , and the velocity profile is assumed to be a typical velocity profile of boulder debris flows over a rigid bed [Ashida et al., 1988], which is also known as the velocity profile of dilatant fluid [Takahashi, 1977] and can be expressed as

$$\frac{\partial u}{\partial z} = \frac{5u_m}{2h} \left(1 - \frac{z}{h} \right)^{1/2} \quad (8)$$

Then, f for debris flows can be obtained by substituting Eqs. (6) and (8) into Eq.(5) to obtain

$$f = \frac{25}{2\rho_m} K(c) \left(\frac{h}{d} \right)^{-2} \quad (9)$$

In contrast, f for turbulent water flows over a rough bed is expressed as

$$f = 2 \left(A_r - \frac{1}{\kappa} + \frac{1}{\kappa} \ln \frac{h}{k_s} \right)^{-2} \quad (10)$$

where A_r is a constant, κ is the von Kármán constant, and k_s the roughness height. In this study, the value of d in each experimental case was used for k_s , assuming that the two were roughly equivalent.

3. RESULTS AND DISCUSSION

3.1 Characteristics of the compiled data

Table 1 summarizes the experiments analyzed

Table 1 Summary of experiments analyzed in the study

Reference	Material	d (mm)	Inclination (deg)	h/d	c (%)	Comparable with c_e ?	Number of cases examined
Takahashi et al. (1996)	sand	2.6, 3.6	16, 18	5.1, 6.6	29, 30	Yes	2
Ashida et al. (1988)	sand, glass beads	2.7–7.8	18–28	3.0–7.4	25–51	Yes	18
Ashida et al. (1986)	pearl clay	0.01	5.8	1100–5300	16–23	No	13
Mainali and Rajaratnam (1994)	sand	0.21–0.42	16	81–229	3–44	No	28
Egashira et al. (1989)	sand, glass beads	0.9–4.1	16–28	4.0–14.1	24–43	Yes	16
Arai and Takahashi (1996)	sand	0.31	17	35, 42	23, 40	No	2
Egashira et al. (1993)	sand	0.16	3, 7	61–77	0–9	Yes	11
Suzuki et al. (2003)	sand	2.2, 2.9	13, 17	6.3–13.2	15–37	Yes	92
Egashira et al. (1990)	sand	1.4, 3.7	0.72–15	6–18	0–24	Yes	21
Total							203

Table 2 Data frequency for sediment concentration (c) in all experiments

c (%)	<0.05	<0.1	<0.2	<0.3	<0.4	>0.4	Total
Relative frequency (%)	8.9	3.9	24.1	39.9	17.2	5.9	100

Table 3 Data frequency for sediment particle size (d) in all experiments

d (mm)	<0.1	<1	<2	<3	<4	>4	Total
Relative frequency (%)	6.4	20.7	6.4	49.8	14.3	2.5	100

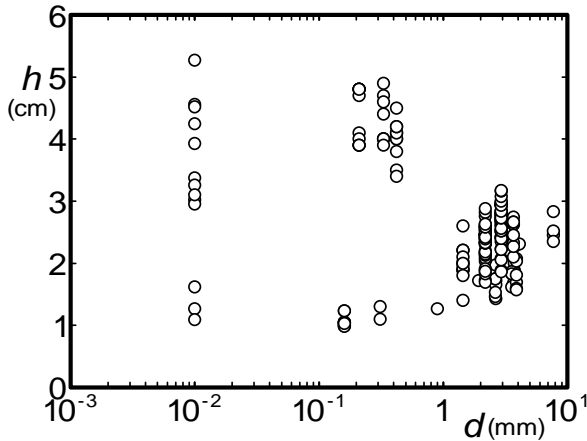


Fig. 1 Relationship between d and h

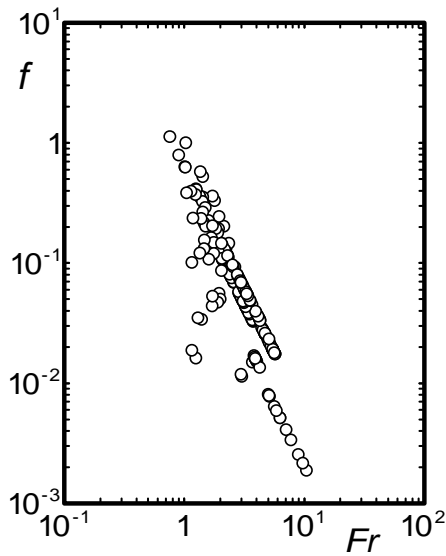


Fig. 2 Relationship between f and Fr

in this study comprising 203 study cases in all. The data frequency for c in all experiments is shown in **Table 2**; the value of c ranged from 0 to 0.55. Considering that the upper limit of c is near that of a close-packed concentration (about 0.6), the range of c values showed sufficient variation for debris flows. Two control experiments without sediment ($c = 0$) by *Egashira et al.* [1993] were in the group of those for $c < 0.05$. **Table 3** shows the data frequency of d in all experiments; the value of d ranged from 0.01 to 7.75 mm. Since experiments to obtain detailed data such as those surveyed in this study are often obtained with flumes shorter than 10 m, the sediment particle size was considered to be restricted to these sizes. **Figure 1** shows the relationship between d and h for the compiled data. Although h did not have a wide variation in values for the same reason as the limitation on d , the range of h/d values was large due mainly to experiments with small particles.

In general, to assure comparability between *in*

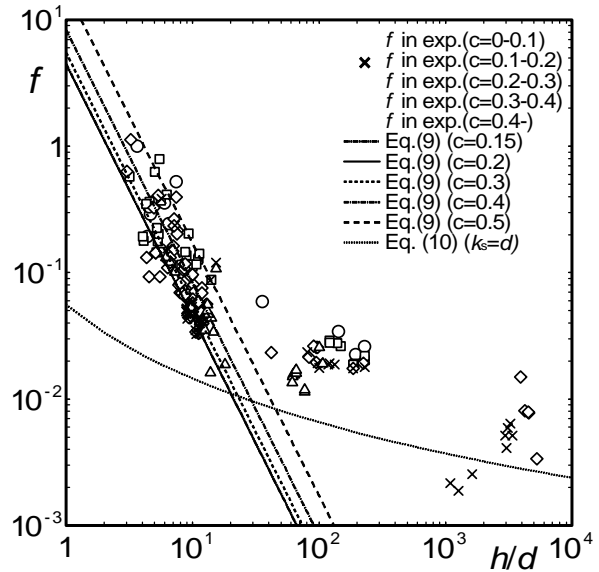


Fig. 3 Relationship between f and h/d for all experiments

situ and experimental debris flows, a similarity of the Froude numbers (Fr) as well as the geometric similarity must exist [*Bando and Mizuyama, 1987; Miyamoto and Itoh, 2003*]. **Figure 2** shows the relationship between f in Eq.(2) and Fr for all experiments. Since the relationship between f and Fr is expressed as

$$f = \frac{2gh \sin \theta}{u_m^2} = 2Fr^{-2} \sin \theta \quad (11)$$

the scattering in **Figure 2** is simply due to the difference in inclination for each wet of experimental conditions. The value of Fr ranged widely, suggesting that experiments included in this study adequately covered various debris flows.

3.2 Relationship between f and h/d

Figure 3 shows the relationship between f and h/d for all the experiments. The data are sorted by the value of c . The theoretical curves of f for both boulder debris flows and turbulent water flows are shown, calculated by Eqs. (9) and (10), respectively. For small h/d values, the f of the experimental data agreed closely with the theoretical line for debris flows. For h/d values greater than about 30, the data diverged from the theoretical line for debris flows and coincided with the theoretical curves of f for turbulent water flows with h/d values between 1000 and 10,000. Thus, the trend of f was roughly explained by the h/d order. The value of c was not biased by h/d , and hyperconcentrated debris flows existed independently of h/d .

Most of the debris flows with smaller h/d values (i.e., < 20) were observed in experiments with typical boulder debris flows [*Ashida et al., 1988; Egashira et al., 1989; Takahashi et al., 1996; Suzuki*

et al., 2003]. However, experiments with sediment-laden flow in which the flow had an upper water layer and a lower debris-flow layer were also included [Egashira *et al.*, 1990]. Some divergent scattering from the theoretical line of f for boulder debris flows (**Fig. 3**) occurred in this range of h/d , possibly caused by conditions different from those assumed for the theoretical line. For example, Ashida *et al.* [1988] and Egashira *et al.* [1989] used glass beads as well as sediment particles as debris-flow materials; these have a different interparticle friction angle and mass density compared to sediment particles. Suzuki *et al.* [2003] showed experimentally using various particle sizes that a bed roughness larger than the sediment particle size affects the flow resistance and results in a greater friction coefficient than predicted by theory. This difference can be explained by particle-collision models similar to the models used to derive the constitutive equations for debris flows [Suzuki and Hotta, 2006]. Another reason for the scattering could be that actual experimental debris flows have a vertical profile for c while Eq.(9) assumes a constant value for c . In addition, the motions of sediment particles do not satisfy the modeled conditions for the constitutive equations in the case of extremely small values of h/d , since the debris flows cannot be assumed as a continuum under that condition. Given that some experimental conditions differed from the assumptions used for the theoretical projections, the experimental data can be considered to show close agreement with the theory.

Debris flows with large h/d values (i.e., 1000–10,000) near the theoretical curves of f for turbulent water flows were observed in experiments with mud flows [Ashida *et al.*, 1986]. Ashida *et al.* [1986] examined the experimental data assuming that interparticle stresses were induced mainly by an electric double layer around the particles. However, the particle size used in their experiments was 0.1 mm, which is much larger than the particle size generally considered to be influenced by the electric double layer (generally less than 0.02 mm). In addition, in their flume tests, a dominant flow layer was in turbulence even though a thin layer was present above the bed in which interparticle stresses were effective, as well as having a plug layer near the surface. Thus, those experimental data were considered as developed turbulent mud flows.

Debris flows with intermediate h/d values (i.e., 30–300) were observed in experiments with turbulent mud flows [Egashira *et al.*, 1993; Mainali and Rajaratnam, 1994; Arai and Takahashi, 1996].

However, the turbulence in these experiments may not have been sufficiently developed. Egashira *et al.* [1993] showed by trace chasing that particle motions become laminar with increasing sediment concentration, and indicated that internal stresses other than the Reynolds stress increase accordingly, resulting in increasing total stress. Debris flows with a higher sediment concentration in the lower layer were included in experiments by Mainali and Rajaratnam [1994]. In particular, they observed far fewer sediment particles in the upper layer during experiments with large particles, inferring that these flows had an interface as well as sediment-laden flows. Furthermore, they confirmed several velocity profiles under different experimental conditions, identifying typical velocity profiles for the dilatant fluid like those shown by Takahashi [1977], as well as log-law-type velocity profiles and intermediate velocity profiles with the shape of lower dilatant and upper log-law profiles. Such an interface was not mentioned by Arai and Takahashi [1996] who also measured velocity profiles. However, the possibility exists that the turbulent structure was not fully developed in the lower layer of their experiment with higher sediment concentration since the turbulence intensity in the lower layer was weaker than that of clear water flow. Hashimoto and Hirano [1995] also conducted flume experiments for turbulent mud flows with h/d values of the same order (around 100) as Arai and Takahashi [1996] and showed that the velocity was distributed linearly in the lower layer for $h/d < 15$, while the upper layer had a log-law distribution. This discontinuous velocity profile was due to the prominent interparticle stresses near the bed.

Thus, debris flows for h/d in the range 30–300 can be considered as intermediate flows between laminar and turbulent debris flows. In other words, a wide transition can occur in debris flows from laminar to turbulent flows. Such transitional debris flows may include debris flows with an upper turbulent flow and a lower immature turbulent / laminar flow.

3.3 Phase transition in debris flows over a rigid bed

How is f derived for transitional debris flows between laminar and turbulent flows such as those investigated in the previous section? Transitional debris flows occasionally have an interface between the upper turbulent and lower laminar layers as described in the previous section citing the experimental results of Hashimoto and Hirano [1995] and Mainali and Rajaratnam [1994]. The

transitional flow is assumed to be as shown in **Figure 4**, based on the same viewpoint as *Takahama et al.* [2000], who analyzed sediment-laden flows. The mean velocity for a debris flow with an upper log-law layer and lower dilatant layer as shown in **Figure 4** is expressed as

$$u_m = \frac{u_1 h_1 + u_2 h_2}{h_1 + h_2} \quad (12)$$

where u_1 and u_2 are the mean velocities for the lower and upper layers, respectively, and h_1 and h_2 are the thicknesses of the lower and upper layers, respectively. The velocities of both the upper and lower layers are connected at $z = h_1$, and u_1 and u_2 can be determined from the constitutive equations for a debris flow and the theoretical velocity profile for a turbulent water flow using Eqs. (2), (9), and (10). The result of substituting Eq.(12) into Eq.(2) with several interface heights for $h = 2$ cm, $c = 0.3$, $\theta = 15$, and $\varphi_s = 38$ is shown in **Figure 5**. The data considered as belonging to transitional flows in the previous section [*Egashira et al.*, 1993; *Mainali and Rajaratnam*, 1994; *Arai and Takahashi*, 1996] and the data of experiments with sediment-laden flows [*Egashira et al.*, 1990] are also shown in **Figure 5**. In the cases in which values of k_s are available [*Egashira et al.*, 1993; *Mainali and Rajaratnam*, 1994], the data are presented for both h/d and h/k_s .

First, the data of *Egashira et al.* [1990] are distributed just above the theoretical line for debris flows. For the relationship between f and h/k_s , the data of *Egashira et al.* [1993] are to the left of the theoretical line for debris flows on the theoretical curve for turbulent water flow. Given that the data of *Egashira et al.* [1993] used in this study indicate a smaller range of concentration (0%–9%) than all the experiments (0%–29%) after filtering by c_e in Eq.(1), the possibility exists that a turbulent structure developed for the total flow layer due to weak interparticle stresses. The f values of *Mainali and Rajaratnam* [1994], however, were distributed above the theoretical curve for turbulent water flow in direct relation to h/k_s . Although the value of k_s was not cited in *Arai and Takahashi* [1996], h/k_s would have been relatively small if f followed the theoretical curve for turbulent water flow. Thus, these transitional data cannot be explained as turbulent flows when compared to k_s , except for the data of *Egashira et al.* [1993].

Next, the changes in variation of f with changing values of h_1/h are discussed. In the phase transition from laminar to turbulent flow, curves of f shift with decreasing h_1/h from the theoretical line for debris flows expressed by Eq.(9) ($h_1/h = 1$) to the

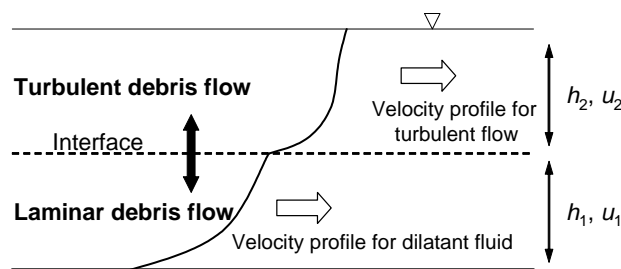


Fig. 4 Definition sketch of the transitional debris flow

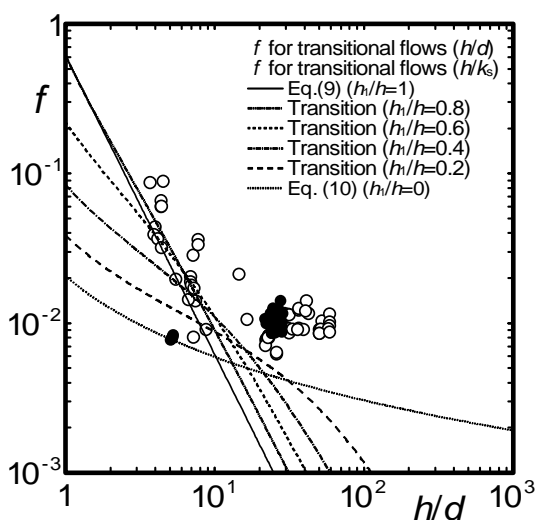


Fig. 5 Relationship between f and h/d for the transitional debris flow

theoretical curve for turbulent water flows expressed by Eq.(10) ($h_1/h = 0$) in **Figure 5**. During the shift, curves of f move above the crossing point of Eqs. (9) and (10) and approach the experimental data for the sediment-laden and transitional flows. This result supports the idea that the phase transition of debris flows from laminar to turbulent flows can be interpreted as a shifting interface that divides the flow structure. However, all transitional debris flows do not always have such a multilayered structure. Furthermore, fully turbulent debris flows may exist that have a value of f greater than the theoretical f value for turbulent water flow, induced by the effect of interparticle stresses. Although the detailed flow structure of debris flows should be examined further, especially in phase transition, the shifting interface is considered an important candidate for the path of phase transition.

4. CONCLUSIONS

The friction coefficients of debris flows over a rigid bed were compared on the basis of h/d under various conditions from a compilation of existing experimental data. The friction coefficients of

debris flows with small h/d values agreed closely with the theoretical value for boulder debris flows derived from the constitutive equations, while the friction coefficients with large h/d values agreed roughly with the theoretical value for turbulent water flows. The friction coefficients of debris flows with intermediate h/d values exceeded the theoretical values for both debris and turbulent water flows. These intermediate debris flows were observed in experiments involving turbulent mud flows. However, an examination of these experiments showed that debris flows in which the turbulent structure was not well developed were possibly included in cases regarded as transitional debris flows from laminar to turbulent flows. In some of the transitional debris flows, an interface dividing the flow structure into an upper turbulent-flow layer and a lower debris-flow layer was observed that was similar to that reported for sediment-laden flows. The friction coefficient for transitional debris flows was modeled considering the shift of this interface. The model friction coefficient was able to explain the value for transitional debris flows, inferring that the phase transition in debris flows is due to the shifting interface.

As a challenge for the future, field observations of the phase transition with a shifting interface in debris flows are needed. Then, the results of this study will provide a simple way to classify actual debris flows, although several problems remain, such as how to determine the representative particle size in the field.

REFERENCES

- Arai, M. and Takahashi, T. (1986): The mechanics of mud flow, Proceedings of the Japan Society of Civil Engineers, No. 375 II-6, pp.69–77. (in Japanese with English summary)
- Arai, M. and Takahashi, T. (1996): A measurement method of fluctuation velocity of high concentration mud-flow and the experimental results, Annual Journal of Hydraulic Engineering, JSCE, Vol. 40, pp.1033–1038. (in Japanese with English summary)
- Ashida K., Yamano, K. and Kanda, M. (1986): Study of hyperconcentrated flow (2) –Mechanism of the flow–, Annals of the Disaster Prevention Research Institute, Kyoto University, No. 29 B-2, pp.3361–3375. (in Japanese with English summary)
- Ashida, K., Egashira, S. and Yajima, H. (1988): Mechanism of debris flow, Annals of the Disaster Prevention Research Institute, Kyoto University, No. 31 B-2, pp.411–422. (in Japanese with English summary)
- Bando, T. and Mizuyama, T. (1987): Procedures of scaled model experiments on the deposition of debris flows, Annual Journal of Hydraulic Engineering, JSCE, Vol. 31, pp.517–522. (in Japanese)
- Crosta, G., Marchetti M., Guzzetti, F. and Reichenbach, P. (1990): Morphological classification of debris-flow processes in South-Central Alps (Italy), Proceedings of the 6th International IAEG Congress, pp.1565–1572.
- Daido, A. (1965): Fundamental study on mud-flow (V), Annals of the Disaster Prevention Research Institute, Kyoto University, No. 8, pp.317–126. (in Japanese with English summary)
- Egashira, S., Ashida, K., Yajima, H. and Takahama, J. (1989): Constitutive equations of debris flow, Annals of the Disaster Prevention Research Institute, Kyoto University, No. 32 B-2, pp.487–501. (in Japanese with English summary)
- Egashira, S., Ashida, K., Takahama, J. and Tanonaka, S. (1990): Sediment transport formula derived from an energy dissipation model of solid–fluid mixture, Annals of the Disaster Prevention Research Institute, Kyoto University, No. 33 B-2, pp.293–306. (in Japanese with English summary)
- Egashira, S., Ashida, K., Tanonaka, S. and Sato, T. (1993): Characteristics of high concentrated sediment laden flow, Annual Journal of Hydraulic Engineering, JSCE, Vol. 37, pp.517–522. (in Japanese with English summary)
- Egashira, S., Miyamoto, K. and Itoh, T. (1997): Constitutive equations of debris flow and their applicability, Proceedings of the 1st International Conference on Debris-Flow Hazards Mitigation, pp.340–349.
- Hashimoto, H. and Hirano, M. (1995): Resistance rule of mud flows on movable and fixed beds, Annual Journal of Hydraulic Engineering, JSCE, Vol. 39, pp.495–500. (in Japanese with English summary)
- Hirano, M., Hashimoto, H., Fukutomi, A., Taguma, K. and Pallu, M.S. (1992): Nondimensional parameters governing hyperconcentrated flow in an open channel, Annual Journal of Hydraulic Engineering, JSCE, Vol. 36, pp.221–226. (in Japanese with English summary)
- Hungr, O., Evans, S.G., Bovis, M.J. and Hutchinson, J.N. (2001): A review of the classification of landslides of the flow types, Environmental & Engineering Geoscience, Vol. 7, No. 3, pp.221–238.
- Itoh, T. and Egashira, S. (1999): Comparative study of constitutive equations for debris flows, Journal of

- Hydroscience and Hydraulic Engineering, Vol. 17, No. 1, pp.59–71.
- Itoh, T., Egashira, S., Miyamoto, K. and Takeuchi, H. (1999): Transition of debris flows over rigid beds to over erodible beds, *Annual Journal of Hydraulic Engineering, JSCE*, Vol. 43, pp.635–640. (in Japanese with English summary)
- Jakob, M. (2005): A size classification for debris flows, *Engineering Geology*, Vol. 79, pp.151–161.
- Mainali, A. and Rajaratnam, N. (1994): Experimental study of debris flows, *Journal of Hydraulic Engineering*, Vol. 120, No. 1, pp.104–123.
- Miyamoto, K. and Itoh, T. (2003): Numerical simulation and similarity of debris flow, *Journal of the Japan Society of Erosion Control Engineering*, Vol. 55, No. 6, pp.40–51. (in Japanese with English summary)
- Naef, D., Rickenmann, D., Rutschmann, P. and McArdell, B.W. (2006): Comparison of flow resistance relations for debris flows using a one-dimensional finite element simulation model, *Natural Hazards and Earth System Sciences*, Vol. 6, pp.155–165.
- O'Brien, J.S. and Julien, P.Y. (1988): Laboratory analysis of mudflow properties, *Journal of Hydraulic Engineering*, Vol. 114, No. 8, pp.877–887.
- Prochaska A.B., Santi, P.M., Higgins, J.D. and Cannon, S.H. (2008): Debris-flow runout predictions based on the average channel slope (ACS), *Engineering Geology*, Vol. 98, pp.29–40.
- Shanmugam, G. (1996): High-density turbidity currents: are they sandy debris flows? *Journal of sedimentary Research*, Vol. 66, No. 1, pp.2–10.
- Suzuki, T. and Hotta, N. (2006): Resistance of the debris flow on the roughness boundary, *Disaster Mitigation of Debris Flows, Slope Failures and Landslides, Proceedings of the INTERPREVENT International Symposium*, pp.129–139.
- Suzuki, T., Hotta, N. and Miyamoto, K. (2003): Influence of riverbed roughness on debris flows, *Journal of the Japan Society of Erosion Control Engineering*, Vol. 56, No. 2, pp.5–13. (in Japanese with English summary)
- Takahama, J., Fujita, Y. and Kondo, Y. (2000): Analysis method of transitional flow from debris flow to sediment sheet flow, *Annual Journal of Hydraulic Engineering, JSCE*, Vol. 44, pp.683–686. (in Japanese with English summary)
- Takahashi, T. (1977): A mechanism of occurrence of mud-debris flows and their characteristics in motion, *Annals of the Disaster Prevention Research Institute, Kyoto University*, No. 20 B-2, pp.405–435. (in Japanese with English summary)
- Takahashi, T. (1978): Mechanical characteristics of debris flow, *Journal of Hydraulic Engineering*, Vol. 104, No. HY8, pp.1153–1169.
- Takahashi, T. (2007): Debris flows: mechanics, prediction and countermeasures, *Taylor and Francis / Balkema*, 448pp.
- Takahashi, T., Nakagawa, H. and Sato, H. (1988): Fan formation due to the runoff of the debris and immature debris flows, *Annual Journal of Hydraulic Engineering, JSCE*, Vol. 32, pp.497–502. (in Japanese)
- Takahashi, T., Satofuka, Y. and Chishiro, K. (1996): Dynamical law of debris flows in inertial regime, *Annals of the Disaster Prevention Research Institute, Kyoto University*, No. 39 B-2, pp.333–346. (in Japanese with English summary)
- Tsubaki, T., Hashimoto, H. and Suetsugi, T. (1982): Grain stresses and flow properties of debris flows, *Proceedings of the Japan Society of Civil Engineers*, No. 317, pp.79–91. (in Japanese)
- Whipple, K.X. (1997): Open-channel flow of Bingham fluids: applications in debris-flow research, *Journal of Geology*, Vol. 105, pp.243–262.
- Winterwerp, J.C., de Groot, M.B., Mastbergen, D.R. and Verwoert H. (1990): Hyperconcentrated sand–water mixture flows over flat bed, *Journal of Hydraulic Engineering*, Vol. 116, No. 1, pp.36–54.